# Influence of Imperfect Carrier Signal Recovery on Performance of SC Receiver of BPSK Signals Transmitted over α-μ Fading Channel

Zlatko J. Mitrović, Bojana Z. Nikolić, Goran T. Đorđević, and Mihajlo Č. Stefanović

***Abstract*—This paper presents the analysis of the reception of binary phase-shift keying (BPSK) signals transmitted over the generalized α-μ fading channel. The selective combining (SC) and then demodulation and detection of the input signal are performed in the receiver while the estimation of the received signal phase is imperfect. We determine the BER dependence on the simultaneous influences of the imperfect reference signal recovery, number of diversity branches, fading severity and average signal-to-noise ratio in the channel.**

***Index Terms*—Diversity systems, Error probability, Fading, Phase-shift keying, Probability density function.**

## I. Introduction

In wireless systems, the variation of instantaneous value of the received signal, i.e. fading of the signal envelope is very common effect, due to the multipath propagation. Fading is one of the main causes of performance degradation in wireless communication systems [1-10].

Diversity technique is certainly one of the most frequently used methods for combating the deleterious effect of channel fading and increasing the communication reliability without enlarging either transmitting power or bandwidth of the channel. The outline of this technique is that the same information is transmitted over few different non-correlated channels. In that way the influence of the fading onto each particular channel is independent. Signals from different channels are, then, combined in order to obtain the resulting signal. In that way the influence of the fading is mainly reduced. Particular diversity methods and combining techniques are presented in [1]-[7]. Selective combining (SC) is combining technique where the strongest signal is chosen among L branches of diversity system.

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The criterion for the selection of the branch is the largest value of instantaneous signal-to-noise ratio among the branches [1]-[3], [6], [7], [10]. Unlike of equal-gain-combining and maximum-ratio-combining techniques, a cophasing in the receiver is not required in SC technique, because, only one branch, one with the best characteristics in that precise moment, is chosen. Although SC technique brings the smallest improvement of receiver performances, the simplicity of practical realization makes the mentioned technique widely spread [1]-[3], [6], [7]. That is the reason why all the calculations for receiver performances in this paper will be presented for SC technique at the reception.

The generalized α-μ fading model was recently proposed in [11], [12] by considering two parameters, namely non-linearity and clustering. The α-μ distribution is written in terms of physically-based fading parameters, namely α and μ, which describe the non-linearity (α) of propagation medium and the multipath wave clustering (μ). This distribution includes the Rayleigh, Nakagami-m, Weibull, and Lognormal distribution as special cases.

Aalo et al. [13] presented a closed-form expression for the average bit error probability for both coherent and noncoherent/differentially coherent binary digital modulations in the generalized Gamma fading channel. Reference [14] considered the performance of linear diversity reception schemes over generalized gamma fading channels under assumption of perfect reference signal extraction.

The phase-locked loop (PLL) is used for carrier signal recovery from non-modulated signal in the receiver. As the receiver is not ideal, a certain phase error appears. The phase error is a difference between the phase of the incoming signal and the phase of the recovered carrier signal in the loop, and this may lead to serious degradation to system performance. It is a statistical process which follow Tikhonov distribution [4], [5], [15]. To the best of our knowledge, the performance of SC receivers of binary phase-shift keying (BPSK) signals in generalized gamma (α-μ) fading in the presence of the imperfect reference signal extraction has not been examined.

In the following, the analysis of the BPSK signal detection over α-μ fading channel is presented. The selective combining of the signals from L branches is performed before the detection. The analysis is performed considering that the carrier signal extraction is imperfect. The analytical expressions for probability density function (PDF) of the signal envelope are determined, as well as the expressions for the average bit error rate (BER) in detection. Using these expressions, the dependence of average BER on average signal-to-noise ratio is obtained for different number of diversity branches L and different standard deviations σφ of phase error. The influence of the α-μ fading parameters on the average BER is determined. Also the graphs which represent the dependence of average BER on $\sigma_\varphi$ are shown.

## II. Model of System

We shall initially introduce a transmitter which sends digitally binary phase-modulated signal in a form $A\cos(\omega_0 t+\Phi_0)$. Depending on a sent symbol, $\Phi_0$ can take following values from the set $\Phi_0 \in \{0,\pi\}$. After the propagation through the fading channel, signal at the k-th branch has the form (Fig. 1):

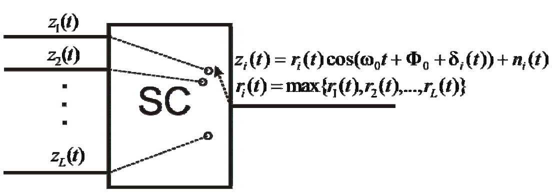
$$z_k(t)=r_k(t)\cos(\omega_0 t+\Phi_0+\delta_k(t))+n_k(t) \tag{1}$$

where $r_k(t)$ is the envelope of the received signal, $\omega_0$ the angular frequency of the carrier, $\Phi_0$ is the transmitted phase of the signal, $\delta_k(t)$ is the random phase (the phase noise caused by multipath fading), and $n_k(t)$ is the additive white Gaussian noise in the $k$-th diversity branch with zero mean value and variance $\sigma^2$. It is assumed that the noise power is same in every diversity branch and fading is uncorrelated among different branches.

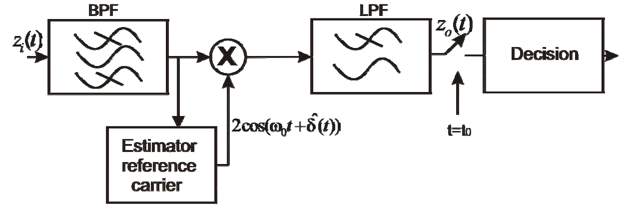
Regarding the above mentioned assumption, the chosen branch in the combining circuit is the one in which the envelope of the received signal has the largest value. As it is shown in Fig. 1, the signal envelope at the output of the combining circuit is:

$$r_i(t)=\max\{r_1(t),r_2(t),\dots,r_k(t),\dots,r_L(t)\}. \tag{2}$$

After the combining, signal is first led to the band-pass filter (BPF) with central frequency $f_0$. The filtered signal is then multiplied by the signal from the estimator of reference carrier. Resulting signal is next led into the low-pass filter (LPF) and sampled in moments $t=t_0$. Finally, on the basis of the sampled value $r_i(t_k)\cos(\Phi_0+\varphi(t_k))$ decision block determines which phase of the signal is transmitted.




a)




b)

Fig. 1. (a) Selection combining and (b) signal detection of BPSK signals.

The purpose of the PLL is to estimate the phase of the incoming signal. In ideal case, the estimated phase should be equal to the phase of the incoming signal $\delta_i(t)$. However, in practical realizations there is certain disagreement between the estimated phase $\hat{\delta}(t)$ and the phase of the signal $\delta_i(t)$. This disagreement is phase error and it is expressed as $\varphi(t)=\delta_i(t)-\hat{\delta}(t)$. The PDF for this phase error corresponds to Tikhonov distribution [4], [5], [15]:

$$p_\varphi(\varphi)=\frac{e^{\alpha_{PLL}\cdot\cos\varphi}}{2\pi\cdot I_0(\alpha_{PLL})},\quad -\pi\le\varphi<\pi \tag{3}$$

where the parameter $\alpha_{PLL}$ represents the signal-to-noise ratio in the PLL circuit and gives the information about the preciseness of phase estimation of incoming signal. It can be assumed $\alpha_{PLL}=1/\sigma_\varphi^2$, where $\sigma_\varphi$ is a standard deviation of the phase error [4], [5], [15]. The modified Bessel function of the first kind and order zero is denoted by $I_0(\cdot)$ [16, eq. (8.406)].

The PDF of the signal envelope at the output of the combining circuit with L branches can be written as [3]:

$$p_{r_i}(r_i)=L\cdot p_r(r_i)\left(\int_0^{r_i}p_r(t)dt\right)^{L-1} \tag{4}$$

where $p_r(r)$ is the PDF of the signal envelope at the $k$-th branch. Since the envelopes of the signals in these branches underlying $\alpha$-$\mu$ distribution with same characteristics the expression (4) can be written as:

$$p_{r_i}(r_i)=L\cdot\frac{\alpha\cdot\mu^\mu\cdot r_i^{\alpha\mu-1}}{\hat{r}_i^{\alpha\mu}\cdot\Gamma(\mu)}\cdot\exp\left(-\mu\frac{r_i^\alpha}{\hat{r}_i^\alpha}\right)\left(\int_0^{r_i}\frac{\alpha\cdot\mu^\mu\cdot t^{\alpha\mu-1}}{\hat{t}^{\alpha\mu}\cdot\Gamma(\mu)}\cdot\exp\left(-\mu\frac{t^\alpha}{\hat{t}^\alpha}\right)dt\right)^{L-1}. \tag{5}$$

After the classical analysis of the signal detection [9], [10], the expression for the conditional BER for BPSK signal, as a function of signal-to-noise ratio in the channel $\gamma=\frac{r_i^2}{2\sigma^2}$, $\sigma^2=\overline{n^2(t)}$ and phase error $\varphi$, can be presented as:

$$P_{e/\varphi,\gamma}=\frac{1}{2}\operatorname{erfc}(\sqrt{\gamma}\cos\varphi). \tag{6}$$

The average BER is:

$$BER=\frac{1}{2}\int_{-\pi}^{\pi}\int_{0}^{\infty}\operatorname{erfc}\left(\sqrt{\gamma}\cos\varphi\right)\cdot L\cdot\frac{\alpha}{2\Gamma(\mu)}\cdot\left(\frac{\Gamma\left(\mu+\frac{2}{\alpha}\right)}{\Gamma(\mu)}\right)^{\frac{\alpha\mu}{2}}\cdot\frac{\gamma^{\frac{\alpha\mu}{2}-1}}{\bar{\gamma}^{\frac{\alpha\mu}{2}}}\cdot$$

$$\cdot\exp\left(-\left(\frac{\gamma}{\bar{\gamma}}\cdot\frac{\Gamma\left(\mu+\frac{2}{\alpha}\right)}{\Gamma(\mu)}\right)^{\frac{\alpha}{2}}\right)\cdot\left(\int_{0}^{\sqrt{\frac{\gamma}{\bar{\gamma}}\frac{\Gamma\left(\mu+\frac{2}{\alpha}\right)}{\Gamma(\mu)}}\mu^{-\frac{2}{\alpha}}}\frac{\alpha\cdot\mu^{\mu}}{\Gamma(\mu)}\cdot u^{\alpha\cdot\mu-1}\cdot e^{-\mu\cdot u^{\alpha}}du\right)^{L-1} \qquad (7)$$

$$\cdot\frac{e^{\alpha_{PLL}\cdot\cos\varphi}}{2\pi\cdot I_0(\alpha_{PLL})}\cdot d\gamma\cdot d\varphi$$

where is $\bar{\gamma}$ the average signal-to-noise ratio, $\gamma$ is the instantaneous signal-to-noise ratio, $\log_2(\cdot)$ is the logarithm to base 2, $\operatorname{erfc}(\cdot)$ is the complementary error function [16, eq. (7.1.2.)], and Γ(.) is the gamma function [16, eq. (8.310/1)].

## III. Numerical Results

Using (7), one can calculate the average BER for $\alpha$-$\mu$ fading channel and discuss performances of the receiver for different values of $\alpha$ and $\mu$ parameters, standard deviation of phase noise, $\sigma_\varphi$, as well as for different number of diversity branches $L$.

The influence of diversity order on the performances of the receiver can be observed from Fig. 2 where dependence of the average BER on average signal-to-noise ratio ($\bar{\gamma}$) is shown for different values of parameter $L$. The $\bar{\gamma}$ is marked as $\gamma_{sr}$ in all figures. With the increase of the diversity order, performances of the receiver improve. However, larger number of diversity branches reduces the additional gain and increases the complexity of the system. Therefore, it is necessary to find a compromise between the performances of the system and its complexity. Power gain is the highest when order of diversity system changes from $L=1$ to $L=2$. For example, in order to obtain the same value of BER=$10^{-4}$, for parameter values α=2.5, μ=1.5, and $\sigma_\varphi$=5°, it is necessary for average signal-to-noise ratio to reach the value of $\bar{\gamma}=19{,}45$ dB for $L=1$, $\bar{\gamma}=11{,}8$ dB for $L=2$, $\bar{\gamma}=9{,}5$ dB for $L=3$, $\bar{\gamma}=8{,}4$ dB for $L=4$, $\bar{\gamma}=7{,}7$ dB for $L=5$, and $\bar{\gamma}=7{,}25$ dB for $L=6$. It can be noticed that the gain exponential declines with the increase of the order of diversity system. In Table 1 calculated power gains are presented in decibels.

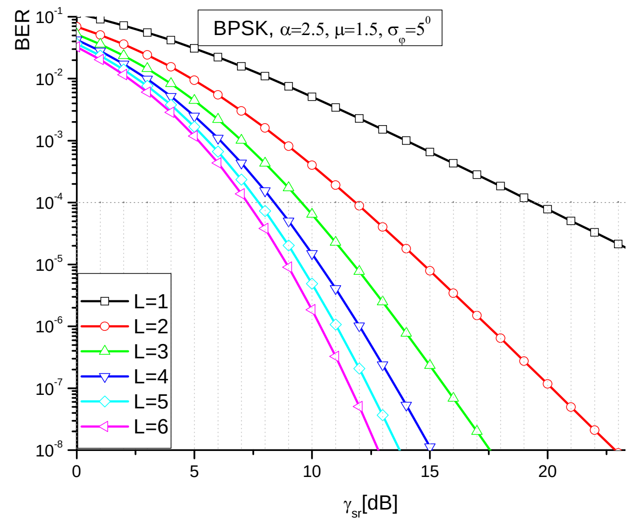



Fig. 2. Influence of diversity order on BER performance.

TABLE I
Gain of the Average Signal-to-Noise Ratio for Different Diversity Orders (for BER=$10^{-4}$)

| Crossing from lower to higher of diversity order L | Gain $\bar{\gamma}$ |
|---|---|
| from L=1 to L=2 | 7,65 dB |
| from L=2 to L=3 | 2,30 dB |
| from L=3 to L=4 | 1,10 dB |
| from L=4 to L=5 | 0,70 dB |
| from L=5 to L=6 | 0,45 dB |

The influence of the carrier extractor quality (the $\sigma_\varphi$ value) on the performances of BPSK receiver is presented in Fig. 3. One can notice that for larger values of $\bar{\gamma}$, the irreducible error floor (BER floor) appears. Therefore, no increase of $\bar{\gamma}$ can cause the BER to fall under the certain value. It is because some of the received bits can be wrongly detected, due to the error in PLL, even when the power of additive Gaussian noise is approaching zero.

The BER dependence on the average signal-to-noise ratio is shown in Fig. 4 for different values of fading parameters $\alpha$ (Fig. 4 (a)) and $\mu$ (Fig. 4 (b)) with diversity order $L=4$ and $\sigma_\varphi=5°$.

Fig. 5 presents the dependence of the BER on the fading parameters α (Fig. 5 (a)) and μ (Fig. 5 (b)) for different values of the average signal-to-noise ratio and constant values of the diversity order $L$=2 and phase noise standard deviation $\sigma_\varphi=5°$.

The dependence of the average BER on the phase noise standard deviation is shown in Fig. 6, while the average signal-to-noise ratio is used as a parameter. In Fig. 6 it can be seen that the curves of the BER dependences on the phase noise standard deviation are approximately constant for the $\sigma_\varphi$ values up to $13^0$.

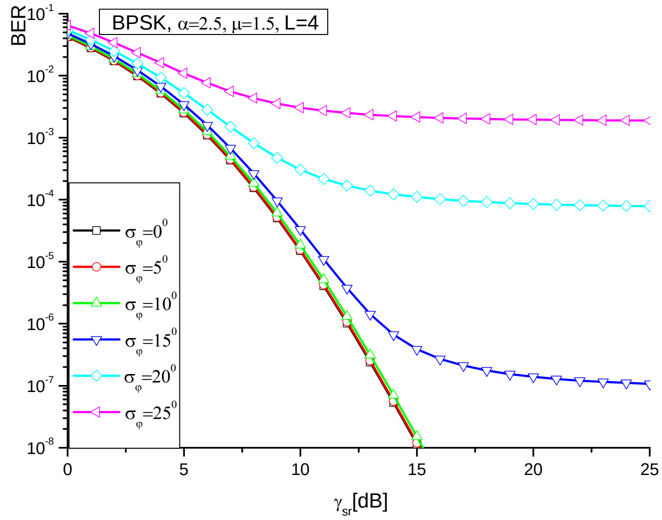
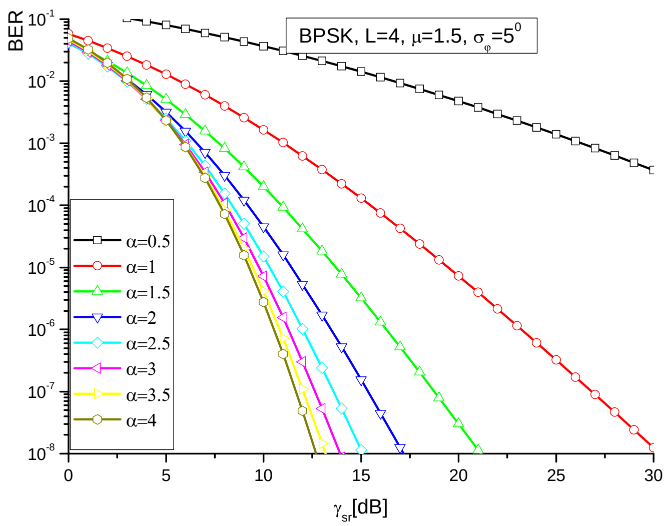
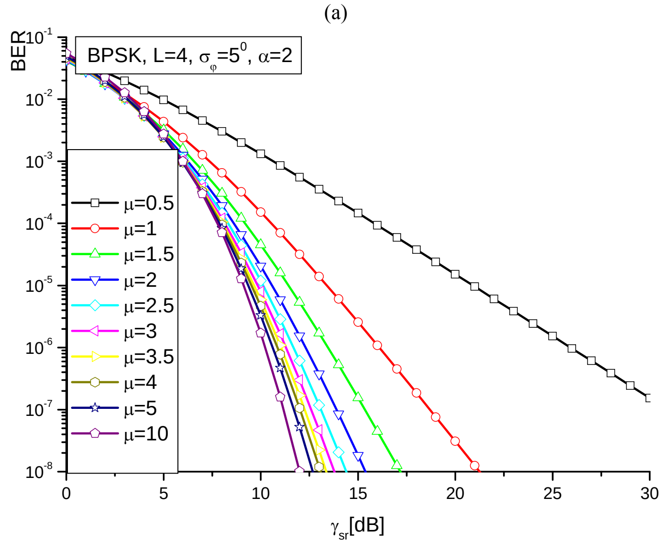



Fig. 3. Influence of carrier extractor quality on BER performance.

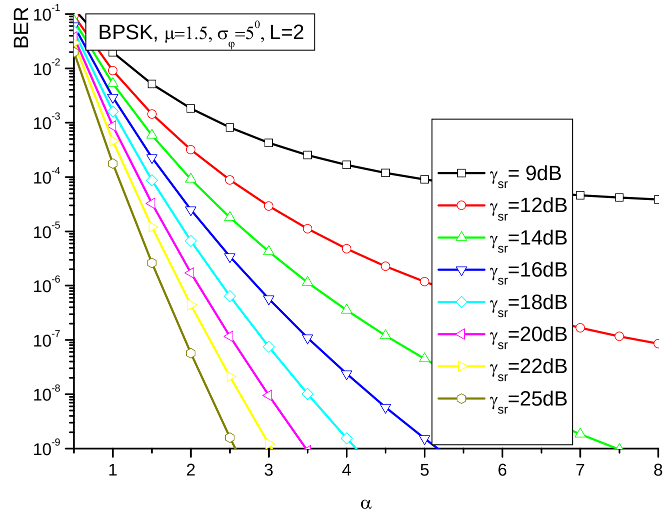



(a)

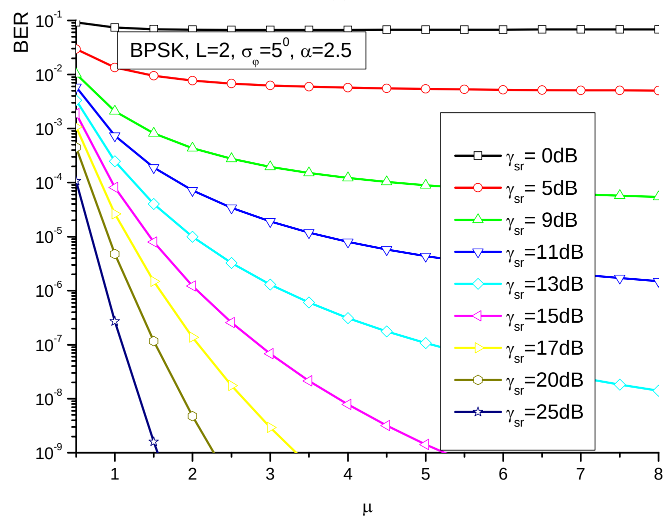



(b)

Fig. 4. Influence of fading parameters: (a) α and (b) μ on the performances of the receiver.




(a)




(b)

Fig. 5. Dependence of average BER on fading parameters α (a) and μ (b) for different values of average signal-to noise ratio.

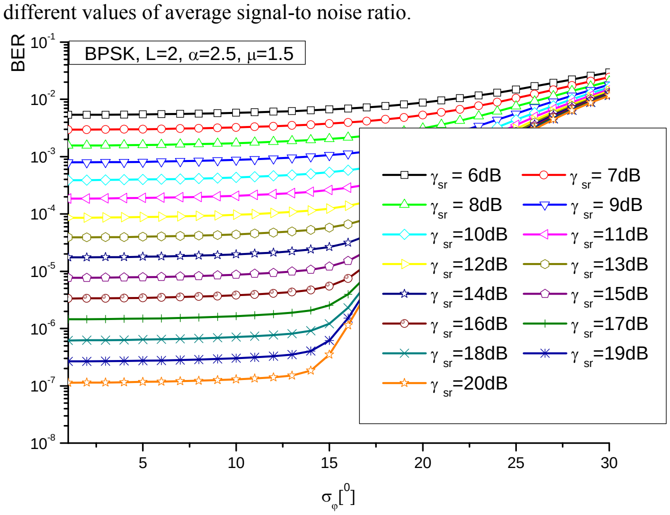



Fig. 6. Dependence of average BER on phase noise standard deviation for different values of average signal-to-noise ratio.

## IV. Conclusion

From the previously performed analysis of selective combining of BPSK signal over α-μ fading channel, the BER is determined in the presence of the imperfect reference carrier extraction. On the basis of presented results it can be concluded in which measure standard deviation of the phase error has the influence on the performances of the receiver. It is shown that the stochastic phase error yields a BER floor. This BER floor is determined for different values of phase error standard deviation. Furthermore, the influence of number of diversity branches on the performances of the system was examined and it is established how much the value of the BER is reduced with the increase of the number of branches. Obtained results enable one to find a compromise between the efficiency (which is measured by the value of BER) and the complexity of the receiver (measured by the number of receiving antennas). More detailed comments on these results are presented in previous part of this paper.

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