The Dependence of Resistivity on Temperature for Thin Superconductors

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Abstract—Measurements made on superconducting very thin layers are analysed by modelization using 2D FEM. The electric field distribution is established and can be seen the differences of this distribution during the superconductive transition. The calculation of the temperature variation of the resistivity is made based on measured temperature variation of the resistance, taking into account also the electro kinetic field distribution.

Index Terms—Superconductivity, very thin layers, resistivity measurements.

I. INTRODUCTION

Measurements made in liquid nitrogen on YBaCuO samples show the transition of high critical temperature superconductors to the superconductive stage. Very thin samples (60...130 microns thickness), tapes of 5-6 mm width and 50-60 mm length are measured using V-A measurement method. To the samples are soldered electric conductors through thin drops of silver, in order to assure good electric contacts for the electric measurements. When measuring the voltage and the current, by their ratio, it can be calculated the resistance of a compound resistor. By modelization and using FEM calculation it is analysed how precise the curve representing the variation of the resistance with the temperature fits to the curve showing the variation of the resistivity of the thin superconductor with temperature.

The transition from normal resistor to superconductor it is produced in a 2-3 K temperature interval, as can be seen in the experimental curve presented in Fig. 1. In between the two temperatures, at the beginning at the end of the transition to the superconductive stage (temperatures marked as $T_a$ and $T_b$) exist the inflexion point of the curve and the temperature corresponding to this point, $T_c$, can be assumed to be the critical temperature of the superconductive layer.

In Fig. 2 is presented the theoretical curve for variation of the resistivity of the high critical temperature superconductor (curve 1) and the variation of the resistivity of silver (2) in the region of 100...120 K. Between the temperatures $T_a$ and $T_b$ the silver’s resistivity has a slight modification, and shall be considered constant in the following calculations [1].

II. THE RESISTANCE OF A COMPOUND RESISTOR

The thin layer of high critical temperature superconductor, together with the small drops of high conductivity silver, represents a compound resistor [4].

The two silver contacts are very important in size because of the small thickness of the superconductive layer [7].

In Fig. 3 is presented the situations of the measurement with only two contact points. It is not convenient to use the same contacts for injecting the current as well as to measure the voltage difference between them [8]. Some of the inconvenient of using this method shall be presented also in this paper.

The four points measurement method can be used for
I

being the resistance of the thin tape of length

through other two electrodes

measured between two electrodes and the current injected
calculating a resistance as the ratio between the voltage

from medium 2 to medium 1, when

temperature between

temperature interval. At a certain moment, corresponding to a

the silver can be considered constant in this very narrow

temperature interval is only of about 2…3 K. The resisitivity of

the tape decreases abruptly in a few seconds, even if the

decomposition of medium 1 is one of silver, and of the tape, are equal one to the other. As the

temperature decreases towards \( T_b \), the resistivity of the tape

becomes smaller and smaller. At \( T_b \) the resistivity of silver is

several times greater than the resistivity of the tape. After a

few seconds the resistivity of the tape is reaching finally the

zero value (superconductive stage of the tape) and remains

constant even if the decreasing of the temperature goes on.

When the current flows through a compound resistor at the

surface separating the two media there are involved some

continuity conditions for the electric field.

The distribution of the electric field in this compound

resistor modifies significantly during the transition from

temperature \( T_a \) (when \( \rho_1 < \rho_2 \)) to \( T_b \) (when \( \rho_1 > \rho_2 \)).

In Fig. 5, it is shown the continuity of the current density

from medium 2 to medium 1, when \( \sigma_1 > \sigma_2 \) (\( \rho_1 < \rho_2 \)). In Fig. 6,

it is shown the situation when the resistivity of medium 2 has

decreased and became smaller than the resistivity of medium 1

(silver contact).

When the thin layer, medium 2, reaches the superconductive

stage the current under the voltage contact has only horizontal

component and the current does not enter inside the silver

voltage contact, medium 1. This situation is presented in

Fig. 7a. Because the current has a finite value inside the layer

2, even if the conductivity \( \sigma_2 = \infty \) for temperatures \( T < T_b \), this

implies that the electric field \( E_2 = 0 \).

In Fig. 7b is presented the situation under the current

injection silver contact. The surface that separates the two

media is equipotential surface.

III. THE NUMERICAL MODELIZATION OF THE COMPOUND RESISTOR

The two measurement methods shown in Figs. 3 and 4 were

modelised in 2D FEM, using QField Terra Analysis Student

version [5]. For the principle of the method, the 250 nodes are

enough to give an image to the students on how measurement

results for calculating the electric resistance can be combined

with numeric modelisation of the compound resistor. Using

both methods the final result (the temperature dependence of

the resistivity of a studied material) can be obtained. Also the

accuracy of the 4 contacts method is proved by the

modelisation also.

As shown in Fig. 2 during the short temperature interval, \( T_a \rightarrow T_b \) the resistivity of the superconductive layer modifies

dramatically while the silver’s resistivity remains practically

the same (we consider it constant in the followings). So, from a

situation when the ratio between the resistivities of the two

materials is smaller than 1, at the temperature \( T_a \),

\( k(T_a) = \rho_1(T_a)/\rho_2(T_a) < 1 \), by reducing the temperature we pass

through a point where the ratio is \( k(T) = \rho_1(T)/\rho_2(T) = 1 \), and we arrive at temperature \( T_b \) where \( k(T_b) = \rho_1(T_b)/\rho_2(T_b) >> 1 \).

Some field distribution representation show the important

difference imposed by the variation of the resistivity. Due to

symmetry only a half of the model was represented. The

representation is made for the situation of the measurement

\[
\begin{align*}
\text{(a)} & \quad J_1 = 0 \quad \overrightarrow{E}_1 = 0 \\
\text{(b)} & \quad J_2 = J_{2n} \quad \overrightarrow{E}_2 = 0 \\
\end{align*}
\]

Fig. 7.
IV. THE CALCULATION OF THE RESISTIVITY CURVE $P_2(T)$

If we assume that the field between the voltage contacts is uniform in each point of the thin layer on the length $l_2$, the curve of the resistivity is obtained by proportionality from the measured curve of the resistance $R_{ms} = f(T)$.

This is valid for greater values of $k = \frac{\rho_1(T)}{\rho_2(T)}$. For $k = 5$, situation presented in Fig. 10, we can see that this condition is nearly fulfilled.

If we take as reference the resistance of the sample measured at $T_b$, one of the last points on the descending resistance curve before reaching the superconductive stage, for this situation $k \approx 50$ and the resistivity can be calculated with

$$\rho_2(T_b) = \frac{R_{ms}(T_b) \cdot S_l}{l_2}. \quad (2)$$

For the analysed sample this value, calculated based on the electric measurements and geometric dimensions of the sample, is $\rho_2(112.75) = 0.102 \text{ nΩ⋅m}$.

At the same temperature the resistivity of silver is $\rho_{Ag}(112.75) = \rho_2(112.75) \approx 5 \text{ nΩ⋅m} [2]$.

The relative value of the resistance measured at another temperature, $T$ in the interval $(T_a, T_b)$, is the resistance $R^*_{ms}$, and their values are calculated in the third row of Table I.

Starting from the situation corresponding to $k = 50$, and using the exact dimensions of the sample (including contacts) several numeric modelizations were made. One of the results of modelization is presented in Fig. 11. Based on the symmetry, only half of the compound resistor was modeled.

The resistance is calculated using the field distribution obtained with QField for each modelization ($k$ from 50 to 0.21) and results are in Table II.

$$R_{md} = \frac{2 \cdot \int E \cdot d\ell}{\int S \cdot dS} = \frac{2V_m}{i} \quad (3)$$

$V_m = 0$ and the value of $V_m$ is read from the program.

The current is calculated, using again the program’s facilities, by integrating on surface $S$.

From the measurements (Table I), we have a dependence on temperature of the relative resistance of the sample.

From modelisation (Table II) we have a dependence of the resistance of the compound resistor on the resistivity of the thin YBCO layer. The reference value in the two tables corresponds to the same situation, the calculated one and the modelized one. For this situation which corresponds to a temperature $T_a = 112.75 \text{ K}$, the resistivity of YBCO is $\rho_2(112.75) = 0.102 \text{ nΩ⋅m}$.

The measured resistance (the relative value) of the compound resistor corresponds to a certain ratio between the resistivity of the silver and the resistivity of the thin YBCO layer. Because was assumed that for the rapid transition to the superconducting stage the resistivity of the silver can be accepted as constant, this means that the relative resistance of the compound resistor is function of the resistivity of the YBCO layer.

<table>
<thead>
<tr>
<th>$k$</th>
<th>50</th>
<th>5</th>
<th>1</th>
<th>0.5</th>
<th>0.33</th>
<th>0.25</th>
<th>0.21</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_2[\text{nΩ⋅m}]$</td>
<td>0.1</td>
<td>1</td>
<td>5</td>
<td>10</td>
<td>15</td>
<td>20</td>
<td>24</td>
</tr>
<tr>
<td>$R^*_{md}[-]$</td>
<td>1</td>
<td>8.82</td>
<td>38.2</td>
<td>76.2</td>
<td>107.8</td>
<td>142.4</td>
<td>170</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$T$ [K]</th>
<th>112.75</th>
<th>113.14</th>
<th>113.78</th>
<th>114.24</th>
<th>114.77</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ms}$ [nΩ]</td>
<td>1.074</td>
<td>21.829</td>
<td>99.401</td>
<td>156.17</td>
<td>169.55</td>
</tr>
<tr>
<td>$R^*_{ms}$ [-]</td>
<td>1.0</td>
<td>20.325</td>
<td>92.552</td>
<td>145.41</td>
<td>157.87</td>
</tr>
</tbody>
</table>
Using (5), based on measured $R_{ms}$, can be obtained the dependence on temperature of the resistivity of YBCO assuming uniform electric field in the layer:

$$
\rho_{ms}(T) = \frac{R_{ms}(T) \cdot S_2}{l_2}.
$$

(5)

Using relations (4), we obtain the dependence on temperature of the resistivity of the YBCO based on modelization and on the dependence on temperature of the relative resistance of the compound resistor, $\rho_{2md}(T)$. Both of them are presented in Table III.

In graphic representation the variation of the resistivity is presented in Fig. 12. It can be seen that exist a difference between the two calculated resistivities.

V. CONCLUSIONS

By the numeric modelization, using QField 2D FEM, the field distribution shows very clearly that the four contact measurement method is much more accurate than the two contact method.

Using the approximation that the electric field inside the thin YBCO layer is uniform, the value of the resistivity results by simple calculation (5), assuming that it is proportional to the resistance calculated from measurements. For very low values of the resistivity of the thin film this is correct, but for medium values and especially for high values of the resistivity this is not correct.

If the resistivity of the silver is much smaller than the resistivity of the YBCO thin layer (at high, “normal”, temperatures) the distance $l’_2$ can be used instead of $l_2$ and the results would be more accurate. This can be seen from the equipotential field lines represented in Fig. 8.

<table>
<thead>
<tr>
<th>$T$ [K]</th>
<th>$\rho_{ms}$ [nΩ·m]</th>
<th>$T$ [K]</th>
<th>$\rho_{ms}$ [nΩ·m]</th>
</tr>
</thead>
<tbody>
<tr>
<td>112.750</td>
<td>0.1</td>
<td>112.75</td>
<td>0.102</td>
</tr>
<tr>
<td>112.906</td>
<td>1.0</td>
<td>113.14</td>
<td>2.079</td>
</tr>
<tr>
<td>113.298</td>
<td>5.0</td>
<td>113.78</td>
<td>9.467</td>
</tr>
<tr>
<td>113.611</td>
<td>10.0</td>
<td>114.24</td>
<td>14.874</td>
</tr>
<tr>
<td>113.916</td>
<td>15.0</td>
<td>114.77</td>
<td>16.148</td>
</tr>
<tr>
<td>114.213</td>
<td>20.0</td>
<td>115.36</td>
<td>16.298</td>
</tr>
<tr>
<td>114.700</td>
<td>22.0</td>
<td>117.08</td>
<td>16.458</td>
</tr>
<tr>
<td>117.982</td>
<td>23.0</td>
<td>118.04</td>
<td>16.698</td>
</tr>
<tr>
<td>120.800</td>
<td>24.0</td>
<td>120.14</td>
<td>16.955</td>
</tr>
</tbody>
</table>

But using $l’_2$ instead of $l_2$ gives unsatisfying results when the two resistivities are of the same range and surely shall give incorrect results for the values of the resistivity when the YBCO approaches to the superconducting stage.

By technical point of view the differences between the two curves of the resistivity are not very significant, but if a very accurate numeric modelization is made the result can be a more accurate curve $\rho(T)$.

Some of the authors [6] when presenting results on thin superconducting film give the dimensions of the cross section of the film (width and thickness) and a result as resistance per unit length, $R’$ [nΩ/m] as function of the temperature, $R’(T)$.

The critical temperature $T_c$ shows to be in fact a little bit smaller but is not modified in a significant manner by the correction proposed by this paper.

REFERENCES


