

Extraction of External Torque Disturbance in Positioning Servomechanism

Milić R. Stojić and Đorđe M. Stojić

Abstract—This paper presents the design of the digitally-controlled positioning servomechanism whose steady state target position is unaffected by arbitrary class of load torque disturbances. For rejection of the torque disturbance effects on the steady state value of motor target angular position, the IMPACT (Internal Model Principle and Control Together) controlling structure is proposed. As an example, the servomechanism is considered in which the output drive torque is produced by using IFOS (Indirect Field Oriented Control) of an induction motor. Some improvements of the controlling structure are proposed in order to eliminate the ringing of control variable and thus to enable easier physical realization of the positioning servomechanism. The theoretical results are confirmed by simulation.

Index Terms—Positioning servomechanism, Load torque disturbance, IMPACT controlling structure.

I. INTRODUCTION

ONE of the key tasks in the design of feedback control systems is to eliminate, or to suppress as much as possible, the influence of external immeasurable disturbances on the steady state value of controlled variable (system output). To perform this task, a number of various controlling structures has been proposed, which can be classified into two main groups: (i) the control schemes that use the IMP (Internal Model Principle), and (ii) the schemes using IMC (Internal Model Control), which are often referred to as the pseudo inverse control.

The IMP based solutions have been proposed in a number of papers [1-5]. Similar task has been performed by using the disturbance observer [6] and a “phase-locked loop” structure [7], for extraction of sinusoidal disturbances. Starting from the idea of Ya.Z. Tsytkin [4], the authors of the survey paper [8] gave the review of previous results based on the application of IMP, focusing on the development of adaptive control for the case of immeasurable time-varying frequency narrow disturbances applied on an active suspension. As a special case, variable frequency sinusoidal disturbances were considered.

A new disturbance observer (DOB) is proposed in [9], is

based on the IMC with the improved performance by modifying the original DOB structure. Furthermore, the proposed observer is adapted to the extended task space formulation. In [10], the state and disturber observer algorithm for linear time-invariant systems is proposed, with the disturbance estimation treated as a plant inversion problem. Since the inverse of a physical system is usually noncausal, the derivatives of output signals, causing that the accurate values of output derivatives become difficult to obtain, which is a common problem for numerous output feedback-disturbance observers. In [10], the tuning parameter is introduced, which can be adjusted to reduce the effects of measuring noise.

Although the disturbance observer, IMP, and IMC based solutions effectively eliminate the influence of immeasurable loads, in general, to a certain degree they suffer from the increased sensitivity to the measuring noise in the feedback signal. In these cases, the problem of measuring noise cannot be completely solved by filtering the feedback signal, since the inclusion of a low-pass digital filter causes an error in the disturbance estimation and rejection of the immeasurable load influence from the feedback control.

In this paper, the IMPACT controlling structure is applied for design of a digitally-controlled positioning servomechanism. The proposed structure eliminates the effects of arbitrary load torque disturbances on the steady state value of the motor target angular position, while keeping the dynamics of the adopted position control loop almost intact. This task is completed by using suitable modification of the original IMPACT structure proposed in [4]. The paper is organized as follows. After the Introduction in Section 1, In Section 2, the IMPACT controlling structure, adopted for the position control of electrical drive with an induction motor, is described. It will be shown that the proposed structure enables that the desired continuous-time set-point transient response of closed-loop system and rejection of load torque disturbance are achieved independently. Section 3 gives a set of simulation results that illustrates the efficiency of the proposed IMPACT structure in rejecting of three typical disturbances from the steady state value of the target angular position of the drive. Section IV considers the problem of ringing of control variable.

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II. IMPACT STRUCTURE OF POSITIONING SERVOMECHANISM

Fig. 1 shows the IMPACT structure primarily proposed by Ya. Z. Tsytkin [4]. Actually, Fig. 1 represents the original IMPACT structure modified for application to the design of position-controlled servo drive with induction motor. Likewise, the modification of general IMPACT structure suitable for control of disturbance invariant processes having large dead-times has been proposed in [11]. In [12], similar control structure has been used in the design and realization of speed-controlled electrical drives.

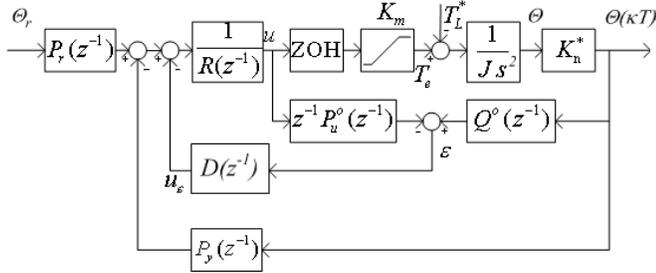


Fig. 1. IMPACT controlling structure of digitally-controlled positioning servomechanism.

The control portion of the structure comprises: two-input nominal plant model, control polynomials $P_r(z^{-1})$, $P_y(z^{-1})$, $R(z^{-1})$, and prediction polynomial $D(z^{-1})$ included into the minor local feedback loop of the structure. The nominal model of the plant consists of the vector-controlled torque driven induction motor and counting-type shaft encoder measuring the motor speed [13]. The encoder generates feedback variable

$$\theta^*(kT) = \frac{K_n}{2\pi} \theta(t) = K_n^* \theta(t) \quad (1)$$

where $\theta(t)$ denotes shaft angular position in radians, T is the sampling period, and K_n is the total number of quantum marks on the encoder disc. Notice that the control portion of the structure in Fig. 1 includes both the two-input nominal plant model explicitly and the model of external disturbance embedded implicitly into the prediction polynomial $D(z^{-1})$.

The zero-hold equivalent nominal model of the plant comprising the vector-controlled induction motor and shaft encoder may be approximated, in the linear regime, by

$$W^0(z^{-1}) = \frac{\theta^*(z^{-1})}{u(z^{-1})} = Z \left[\frac{K_m K_n^* (1 - e^{-Ts})}{J s^3} \right] \quad (2)$$

wherefrom one obtains

$$W^0(z^{-1}) = \frac{z^{-1} P_u^0(z^{-1})}{Q_0(z^{-1})} = C_m \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^2} \quad (3)$$

where $C_m = K_m K_n^* T^2 / 2J$ is the synthetic plant parameter and K_m , K_n^* , and J denote respectively the electromagnetic torque coefficient, number of quantum marks per radian, and motor inertia. Hence, in the IMPACT structure of Fig. 1, $z^{-1} P_u^0(z^{-1}) = C_m z^{-1}(1+z^{-1})$ and $Q^0(z^{-1}) = (1-z^{-1})^2$.

For minimal phase plants, the proper choice of polynomial $R(z^{-1})$ is $R(z^{-1}) = P_u^0(z^{-1}) = C_m(1+z^{-1})$ [11, 12]. In the nominal case, $P_u(z^{-1}) = P_u^0(z^{-1})$, $Q(z^{-1}) = Q^0(z^{-1})$ and for $R(z^{-1}) = P_u^0(z^{-1})$, the closed-loop transfer function $\theta^*(z^{-1})/\theta_r(z^{-1})$ is easily derived from Fig. 1 as

$$\frac{\theta^*(z^{-1})}{\theta_r(z^{-1})} = \frac{z^{-1} P_r(z^{-1})}{Q^0(z^{-1}) + z^{-1} P_y(z^{-1})}. \quad (4)$$

After approximation of torque disturbance $T_L(t)$ by $T_L^*(t) = T_L(t)$ for $kT \leq t < (k+1)T$, $k = 0, 1, 2, \dots$, the system closed-loop transfer function $\theta^*(z^{-1})/T_L^*(z^{-1})$ becomes

$$\frac{\theta^*(z^{-1})}{T_L^*(z^{-1})} = \frac{Q^0(z^{-1}) [1 - z^{-1} D(z^{-1})]}{Q^0(z^{-1}) + z^{-1} P_y(z^{-1})} W_L(z^{-1}) \quad (5)$$

where

$$W_L(z^{-1}) = Z \left[\frac{K_n^*}{J} \frac{1 - e^{-Ts}}{s^3} \right] = \frac{K_n^* T^2}{2J} \frac{z^{-1}(1+z^{-1})}{(1-z^{-1})^2} \quad (6)$$

A. Rejection of disturbance

From Eqs. (5) and (6) and $Q^0(z^{-1}) = (1-z^{-1})^2$, the steady-state error in the presence of a known class of external disturbances $T_L^*(t)$ will become zero, after a certain finite number of sampling periods, i.e.,

$$\lim_{z \rightarrow 1} (1-z^{-1}) \frac{[1 - z^{-1} D(z^{-1})]}{Q^0(z^{-1}) + z^{-1} P_y(z^{-1})} \frac{K_n^* T^2 z^{-1}(1+z^{-1})}{2J} T_L^*(z^{-1}) = 0 \quad (7)$$

Since

$$\lim_{z \rightarrow 1} \frac{1}{Q^0(z^{-1}) + z^{-1} P_y(z^{-1})} \frac{K_n^* T^2 z^{-1}(1+z^{-1})}{2J} \neq 0 \quad (8)$$

equation (7) is satisfied if

$$\lim_{z \rightarrow 1} (1-z^{-1}) [1 - z^{-1} D(z^{-1})] T_L^*(z^{-1}) = 0. \quad (9)$$

Suppose that the class of load torque disturbances is known

and given by its z -transform $T_L^*(z^{-1}) = A(z^{-1})/B(z^{-1})$. Then Eq. (9) is satisfied and consequently the disturbance is completely rejected in the steady-state if

$$1 - z^{-1}D(z^{-1}) = B(z^{-1}) \quad (10)$$

wherefrom one obtains the prediction polynomial

$$D(z^{-1}) = \frac{1 - B(z^{-1})}{z^{-1}}. \quad (11)$$

For example, for the constant, ramp, parabolic, and sinusoidal ($T_L(t) = \sin \omega t$) torque disturbances, the denominator of disturbance model $B(z^{-1})$ equals $1 - z^{-1}$, $(1 - z^{-1})^2$, $(1 - z^{-1})^3$, and $1 - 2z^{-1} \cos \omega T + z^{-2}$, respectively. Furthermore, the disturbance polynomial $B(z^{-1})$ can also be determined for the cases of more complicated disturbances. For example, for the composed disturbance of superposed ramp and sinusoidal signals, the disturbance model $B(z^{-1}) = (1 - z^{-1})^2(1 - 2z^{-1} \cos \omega T + z^{-2})$ should be used for the IMPACT structure design. In such way, for any kind of single or more complicated class of disturbances, the corresponding prediction polynomial $D(z^{-1})$ can be immediately determined by using Eq. (11). Nevertheless, torque disturbance $T_L(t)$ is usually slow varying and therefore the implementation of disturbance polynomial $B(z^{-1}) = (1 - z^{-1})^2$ or the corresponding prediction polynomial $D(z^{-1}) = 2 - z^{-1}$, which corresponds to extraction of ramp disturbances, will effectively reject the influence of torque disturbance on the steady state value of the angular target position of the motor shaft. Moreover, the use of $B(z^{-1}) = (1 - z^{-1})^2$ for calculation of prediction polynomial $D(z^{-1})$ by Eq. (11) will strongly suppress low frequency stochastic disturbances that can be generated by double integration of white noise.

B. Parameter setting

Control polynomials $P_r(z^{-1})$ and $P_y(z^{-1})$ in the main control loop of the structure in Fig. 1 may be determined by the specified closed-loop system pole spectrum or by the desired closed-loop system transfer function. Since the closed-loop system is of the second order, the set-point closed-loop continuous system response may be specified by the closed-loop system transfer function

$$\frac{\theta(s)}{\theta_r(s)} = \frac{\sigma^2}{(s + \sigma)^2} \quad (12)$$

which guarantees a strictly aperiodical and sufficiently fast set-point response.

Let us suppose the desired bandwidth of the closed-loop system $f_c \approx 6$ Hz. Then, with sampling period $T = 0.01$ s, one calculates $e^{-\sigma T} = e^{-2\pi f_c T} = 0.6859$ and $\sigma = 37.7$.

The zero-hold equivalent pulse transfer function of (12) is

$$\frac{\theta(z^{-1})}{\theta_r(z^{-1})} = Z \left[\frac{1 - e^{-\sigma T}}{s} \frac{37.7^2}{(s + 37.7)^2} \right] = \frac{0.05549z^{-1} + 0.04316z^{-2}}{1 - 1.3720z^{-1} + 0.4705z^{-2}}. \quad (13)$$

After substituting $Q^0(z^{-1}) = (1 - z^{-1})^2$ into (4) and then equating identically Eqs. (4) and (13), one determines control polynomials $P_r(z^{-1})$ and $P_y(z^{-1})$ as

$$P_r(z^{-1}) = 0.05549 + 0.04316z^{-1} \quad (14a)$$

$$P_y(z^{-1}) = 0.6280 - 0.5295z^{-1}. \quad (14b)$$

III. SIMULATION

In order to examine the efficiency of the IMPACT structure in extraction of load torque disturbances, several simulation runs have been carried out. In the considered positioning servomechanism [14], the employed induction motor has inertia of $J = 0.0459$ kg·m². The incremental encoder giving 2500 pulses per revolution is used. The synthetic parameter $C_m = K_m K_n^* T^2 / 2J = 0.025$ is adopted. Note that this parameter can be easily measured on an experimental setup. Thus, other parameters required for the simulation are: $K_n^* = 2500 / \pi = 397.887$, $K_m = 2C_m J / K_n^* T^2 = 0.05768$, and $K_m K_n^* / J = 500$. In all simulation runs that follow, the disturbance model polynomial $B(z^{-1}) = (1 - z^{-1})^2$ and the corresponding prediction polynomial $D(z^{-1}) = 2 - z^{-1}$ calculated from (11) are used. Recall that these polynomials correspond to extraction of the constant, ramp, and slow varying disturbances.

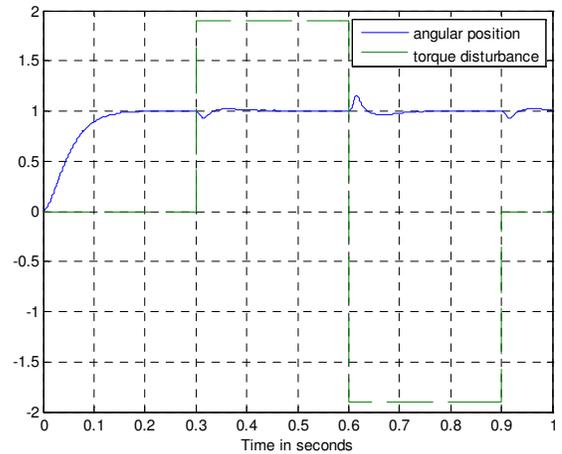


Fig. 2. Extraction of constant step disturbance.

In the first simulation run, two successive step constant torque disturbances are applied. Fig. 2 shows that the influence of the disturbance on the steady state value of target position is completely eliminated after relatively brief transient responses.

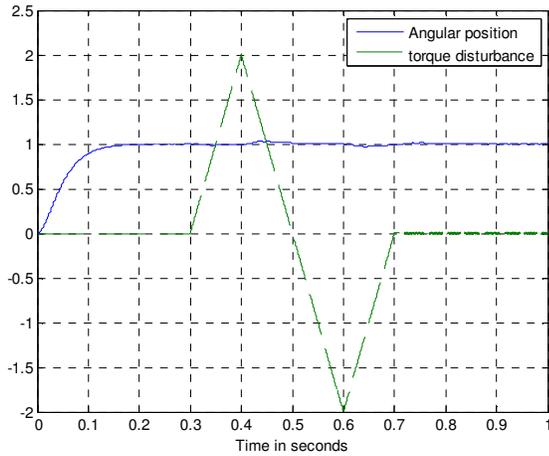


Fig. 3. Extraction of ramp disturbances.

In the second simulation run, the combined ramp torque disturbance shown in Fig. 3 is applied. Fig. 3 shows that the controlling structure completely rejects the disturbance from the steady state value of the servomechanism angular position.

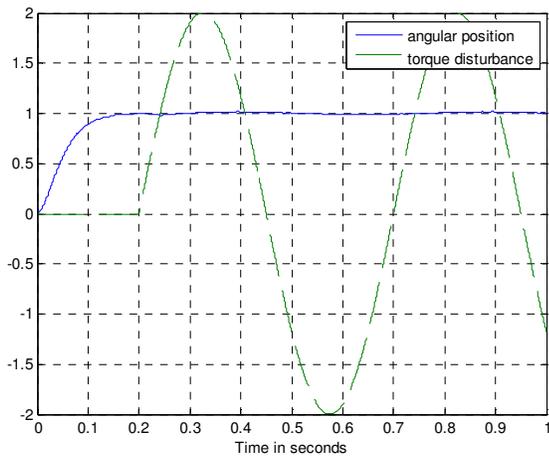


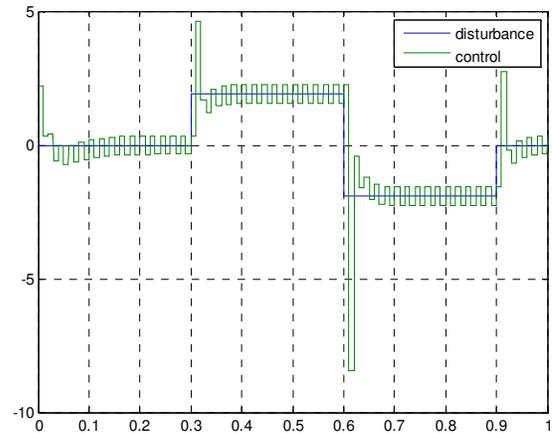
Fig. 4. Extraction of sinusoidal disturbance.

Finally, the sinusoidal torque disturbance shown in Fig. 4 is applied. Fig. 4 illustrates the efficiency of the proposed IMPACT structure in extraction of this kind of external disturbances. Namely, Fig. 4 shows that the disturbance practically does not affect the steady state target position of the positioning servomechanism.

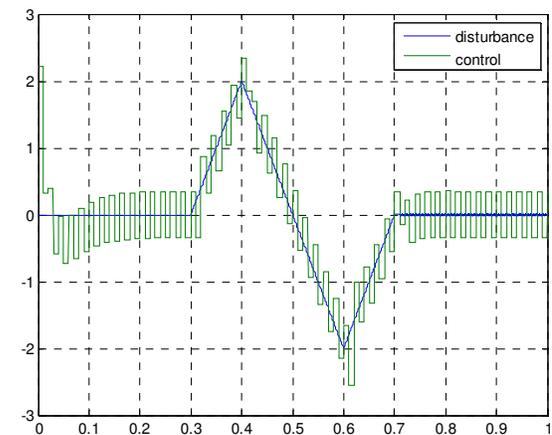
IV. RINGING OF CONTROL

Figs. 2, 3 and 4 show the efficiency of the IMPACT controlling structure in rejection of different external torque disturbances from the steady state target position of the servomechanism. Nevertheless, the physical realization of the servomechanism requires the additional analysis of the shape

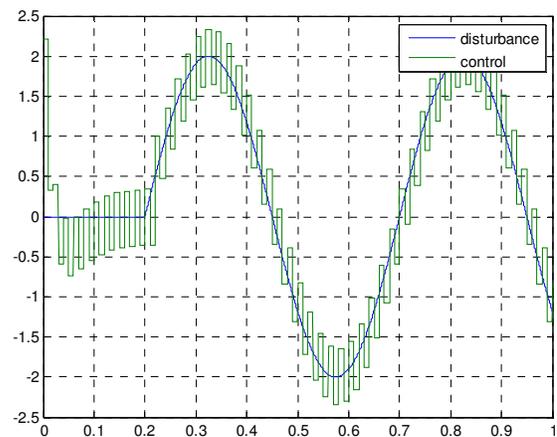
and amplitude of control variable. To this end, in Fig. 6 the disturbances and their controls are shown. From the figure, it is seen that in all cases the controls exhibit fluctuations that are often called “ringing of control”, which could produce serious difficulties in practical realization of driving inverter.



(a)



(b)



(c)

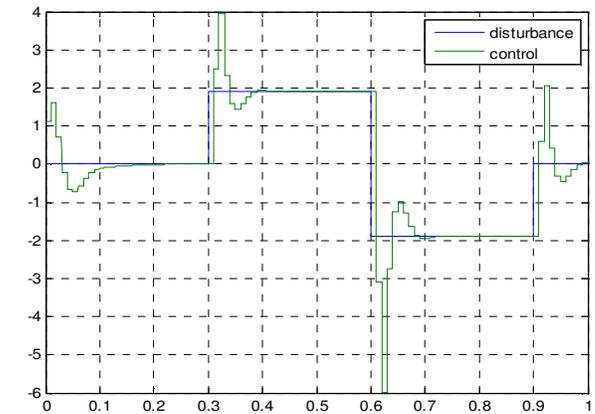
Fig. 5. Ringing of control variables for different types of torque disturbances: (a) constant, (b) ramp, and (c) sinusoidal.

The fluctuations arise due to the present of pole $z = -1$ in the cascade filter $R(z) = z/0.025(z+1)$ within the control portion of the IMPACT structure. This pole generates periodical fluctuations of control variable, having the period equal to $T/2$. To prevail over these difficulties, one can use the idea of Dahlin, which consists in eliminating the critical pole, and thus the ringing of control, by setting $z=1$ in $R(z)$ to obtain $R(1) = 1/0.05$. In doing so, the new structure similar to the IMPACT is obtained. The structure was simulated and the results of simulation runs are shown in Fig. 5. By comparing Figs. 4 and 5, it is noticed that the ringing of control does not exist anymore.

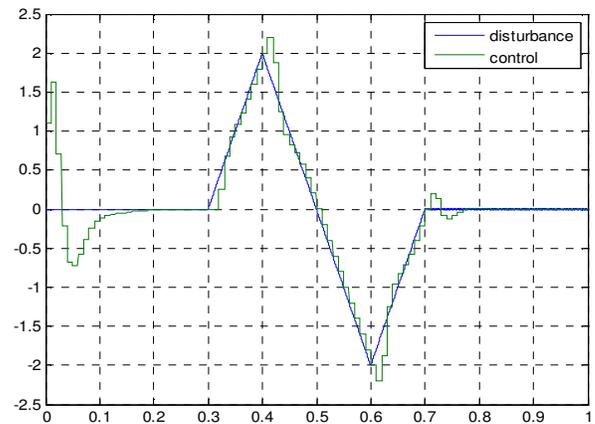
After substitution of $R(z) = z/0.025(z+1)$ by $R(1) = 1/0.05$, the obtained controlling structure is not the proper IMPACT structure any longer and therefore its quality of disturbance transient response and ability of extraction of external disturbances are reduced. This is illustrated by simulation of the new structure and simulation results are given in Figs. 7, 8, and 9. Comparing Figs. 2, 3, and 4 with the corresponding Figs. 7, 8, and 9, one can conclude that, in the new control structure, the setting time and overshoot of disturbance transient response prolongs and increases, respectively. Furthermore, the ability of the new structure in rejection of torque disturbances is slightly reduced. Namely, in the steady state, a small amount of disturbances is noticed. However, this disadvantage of the modified IMPACT structure does not disqualify its practical application in designing of positioning mechanisms.

V. CONCLUSION

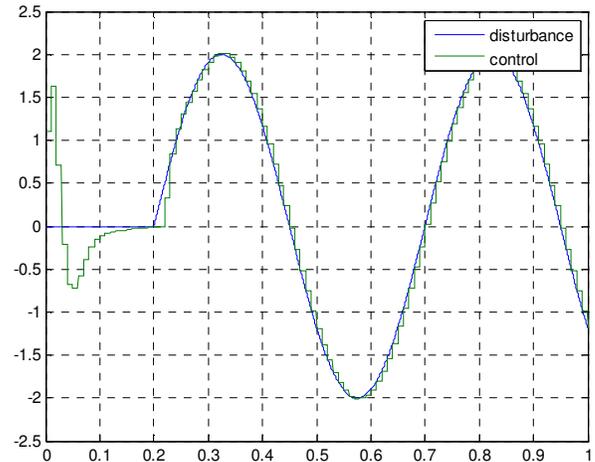
One of the tasks of a control system is that in the presence of external disturbances it tracks the reference signal without steady-state error. In most practical applications, some a priori information about the class of disturbances is available. In such cases, extraction of immeasurable external disturbance is possible by using the IMPACT controlling structure proposed in this paper. The structure has been applied for design of positioning servomechanism in which the steady state target angular position of motor shaft is invariant in respect of external load torque disturbances. For any kind of known class of disturbance it is possible to design the controlling structure that completely rejects or suppresses the influence of disturbance on the steady state value of motor angular position. The simulation results confirm the ability of the structure for disturbance extraction. For the sake of clarity, the procedure outlined in this paper is illustrated by the design of disturbance invariant positioning servomechanism in which the control plant includes a vector controlled induction motor. It is of particular interest to note that the procedure stays unchanged if some other type of driving motor is used. The only difference is the inclusion of the corresponding nominal plant model.



(a)



(b)



(c)

Fig. 6. Control variables without ringing for different types of torque disturbances: (a) constant, (b) ramp, and (c) sinusoidal.

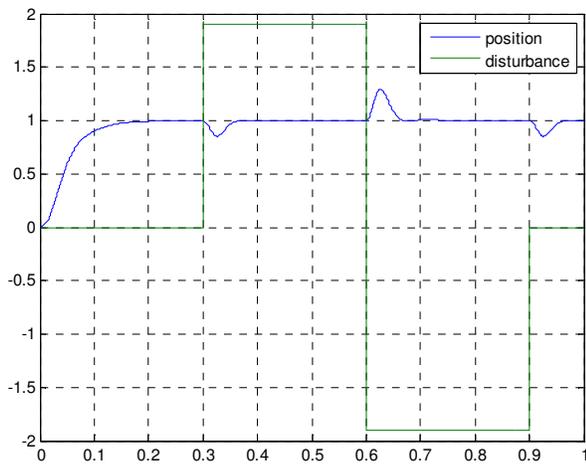


Fig. 7. Extraction of constant step disturbances.

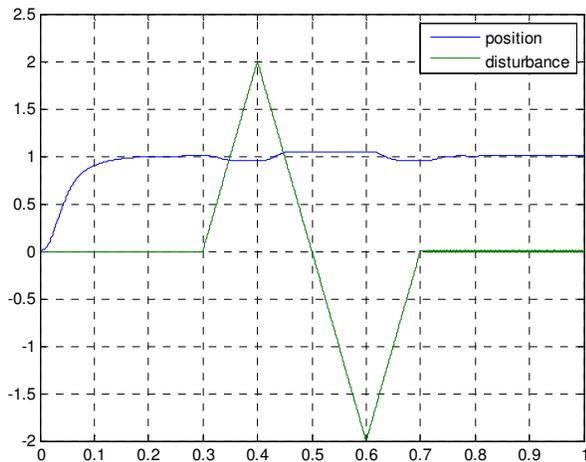


Fig. 8. Extraction of ramp disturbance.

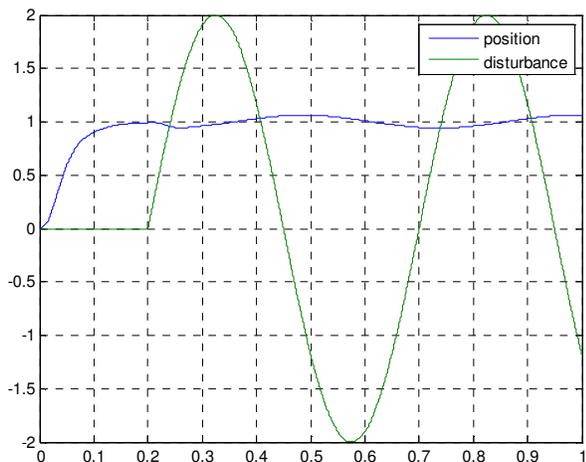


Fig. 9. Extraction of sinusoidal disturbances.

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