Robust RST Controller Design by Convex Optimization

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***Abstract*—A convex optimization-based robust RST controller design approach is proposed in this paper. It is shown that the system specifications given as the Nyquist robust stability criteria, absorption of effects for different types of external disturbances and the pole placement problem for LTI systems can be transformed into convex constraints. The controller design problem is then reduced to solving the standard quadratically-constrained convex program. The analysis also illustrates the potential limitations of the method through inherent tradeoff between robustness stability and performance specifications. The design procedure is illustrated on a practical industrial problem, showing that the proposed method can provide very robust solutions with good performance.**

***Index Terms*—Control design, Digital control, Optimization**

I. INTRODUCTION

General procedure to design and tune a good controller is [1]: I) To specify the desired control-loop performances; II) To obtain a dynamic model of the plant to be controlled (e.g. from real data by identification); III) To develop a suitable controller design methodology, compatible with the desired performances and the corresponding plant model; IV) To have a procedure for controller validation and onsite re-tuning; V) To develop and implement software packages with real-time capabilities for data acquisition, system identification, control design and on-site commissioning. Energy and material savings as well as improvement in the quality of the products should be a result of a well designed control system.

This paper presents a new approach for the design of the RST controller, which can be shown to cover all the linear control laws for linear SISO system [1]. In previous work, the pole placement methodology for synthesis of linear SISO systems produced the RST controller in the final step [2] as a solution of one or more of the Diophantine equations [2]-[5]. However, Diophantine equations do not have a unique solution, and different possible solutions of the RST controller parameters have different implications related to the control objectives [2],[3]. To choose the RST polynomials that better fit the control system requirements can be a very difficult numerical problem, especially in auto- and self-tuning control systems. Because of these difficulties general RST controller design for industrial applications is still a challenge [7],[5].

In this paper, we develop a procedure for design of robust RST controllers based on the use of convex optimization. We develop methods to turn the Diophantine equations and robust stability specifications into convex constraints, and formulate the RST controller problem as a quadratically-constrained convex feasibility problem that can be solved very efficiently on regular compute hardware. We also utilize the absorption principle to specify the control objectives for the steady-state tracking trajectory in the presence of disturbances. The methodology is illustrated through an example of the controller design for the flexible coupled motor servo drive with load [12].

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II. PROBLEM FORMULATION AND DESIGN STEPS

In the RST control structure shown in Fig.1, the plant is described by its pulse transfer function or by polynomials $B(z^{-1})$ and $A(z^{-1})$, the control structure is given by polynomials $R(z^{-1})$, $S(z^{-1})$ and $T(z^{-1})$, r is the reference signal, and signals d and v model the influence of external disturbances and noise on the system output y.

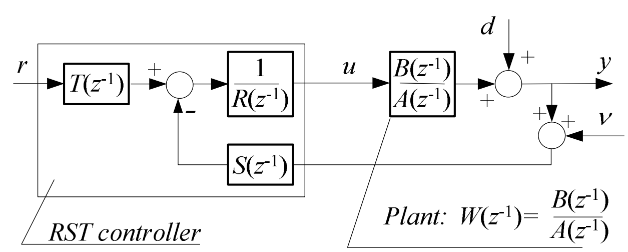

Fig.1 RST control structure.

The nominal model of the plant in Fig.1 is given by

$$W^0(z^{-1}) = \frac{B^0(z^{-1})}{A^0(z^{-1})} \cdot \qquad (1)$$

The uncertainties of the plant modeling may be adequately described by a multiplicative bound $\alpha(\omega)$ [2],[6]

$$W(z^{-1}) = W^0(z^{-1})\left(1+\delta W(z^{-1})\right) \tag{2}$$

$$\left|\delta W(e^{-j\omega T})\right| \le \alpha(\omega),\ \ \omega \in \left[0, \pi/T\right], \tag{3}$$

where $W(z^{-1})$ represents the actual plant behavior.

Under the nominal conditions ($B(z^{-1}) \equiv B^0(z^{-1})$ and $A(z^{-1}) \equiv A^0(z^{-1})$), the closed-loop transfer functions $y(z^{-1})/r(z^{-1})$, $y(z^{-1})/d(z^{-1})$ and $y(z^{-1})/\nu(z^{-1})$ are easily derived from Fig. 1 as

$$G_{ry}(z^{-1}) = \frac{y(z^{-1})}{r(z^{-1})} = \frac{T(z^{-1})B^0(z^{-1})}{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})} \tag{4}$$

$$G_{dy}(z^{-1}) = \frac{y(z^{-1})}{d(z^{-1})} = \frac{A^0(z^{-1})R(z^{-1})}{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})} \tag{5}$$

$$G_{\nu y}(z^{-1}) = \frac{y(z^{-1})}{\nu(z^{-1})} = -\frac{B^0(z^{-1})S(z^{-1})}{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})}. \tag{6}$$

The system set-point response can be adjusted according to desired system closed-loop transfer function (pole placement problem [2])

$$G_{de}(z^{-1}) = \frac{y(z^{-1})}{r(z^{-1})} = \frac{B_{de}(z^{-1})}{A_{de}(z^{-1})}. \tag{7}$$

Namely.

$$\frac{T(z^{-1})B^0(z^{-1})}{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})} = \frac{B_{de}(z^{-1})}{A_{de}(z^{-1})}\frac{A_0(z^{-1})}{A_0(z^{-1})} = \frac{N_{pol}(z^{-1})}{K_{pol}(z^{-1})}, \tag{8}$$

where $A_0(z^{-1})$ is the so-called observer polynomial [2]. The condition of the desired dynamical behavior of the nominal closed-loop system is then given by Diophantine equation

$$A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1}) \equiv K_{pol}(z^{-1}). \tag{9}$$

In order to specify the desired steady-state behavior of the system, according to Fig. 1, we use the absorption principle [6] and derive the error signal as

$$e = r - y = r - \frac{TB^0}{A^0R + B^0S}r - \frac{B^0S}{A^0R + B^0S}\nu - \frac{A^0R}{A^0R + B^0S}d \tag{10}$$

where, for the sake of brevity, variable arguments are omitted from notations of variables.

The absorption principle has the intention to embed the disturbance model into the control algorithm in order to suppress or reject the influence of the disturbance on the steady-state value of the process output. In the case of regular disturbances belonging to a certain class [6]:

$$f \in \left\{\Phi(z^{-1})f(k) = 0,\ \text{for}\ \ k \ge (\deg \Phi)\right\}, \tag{11}$$

we will call the polynomial $\Phi(z^{-1})$ the absorbing polynomial. The absorption filter $\Phi(z^{-1})$ is designed for a known class of disturbances and can be simply resolved as [3]

$$\Phi(z^{-1}) = D_w(z^{-1}),\ \ \text{from}\ \ \ f(z^{-1}) = \frac{N_w(z^{-1})}{D_w(z^{-1})}. \tag{12}$$

In the case of a stochastic disturbance $d(t)$, the absorption filter should suppress as much as possible the effects of the disturbance on the system output. Thus, for a low frequency disturbance $d(t)$, which can be generated by double integration of the white noise, an appropriate choice of the absorption filter is $\Phi_d(z^{-1}) = (1-z^{-1})^2$ that corresponds to absorption of a linear (ramp) disturbance [6].

The absorption conditions of the steady-state influence of the external disturbance $d(t)$ and the reference input $r(t)$ on the error tracking system signal (10) are derived from (11) and (10) as

$$A^0(z^{-1})R(z^{-1}) - \Phi_d(z^{-1})M_d(z^{-1}) \equiv 0 \tag{13}$$

$$B^0(z^{-1})T(z^{-1}) + \Phi_r(z^{-1})M_r(z^{-1}) \equiv K_{pol}(z^{-1}) \tag{14}$$

Then, the system of Fig. 1 satisfies the condition of robust stability if the nominal plant is stable and the following inequality holds [2]6

$$\alpha(\omega) \le \left|\frac{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})}{B^0(z^{-1})S(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]}. \tag{15}$$

By taking into account (9) the condition of robust stability (15) can be transformed to

$$\left|S(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]} \le \frac{1}{\alpha(\omega)}\left|\frac{K_{pol}(z^{-1})}{B^0(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]} \tag{16}$$

Suppression of disturbance and noise effects on the system output can also be expressed by relations

$$\begin{aligned}\left|\frac{A^0(z^{-1})R(z^{-1})}{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]} &\le \\ \le \left|G_{dy}^{desired}(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]}&\end{aligned} \tag{17}$$

$$\begin{aligned}\left|\frac{B^0(z^{-1})S(z^{-1})}{A^0(z^{-1})R(z^{-1}) + B^0(z^{-1})S(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]}& \\ \le \left|G_{\nu y}^{desired}(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]}&\end{aligned} \tag{18}$$

where

$$\left|G_{dy}^{desired}(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]},\ \left|G_{\nu y}^{desired}(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\ \omega T\in[0,\pi]}$$

are the desired or required transfer functions whose magnitudes at all frequencies should be as small as possible. From (18), the condition of noise suppression can be expressed as

$$\left|S(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\,\omega T\in[0,\pi]} \le \left|G_{vy}^{desired}\right| \left|\frac{K_{pol}(z^{-1})}{B^0(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\,\omega T\in[0,\pi]} . \tag{19}$$

Given the above formulation, the controller design procedure can be formulated in the following steps: 1) Define the desired characteristic polynomial $K_{pol}(z^{-1})$, frequency function $G_{vy}^{desired}(\omega)$ $\omega\in[0,\pi/T]$, and the desired absorption filters $\Phi_r(z^{-1})$ and $\Phi_d(z^{-1})$ based on *a priori* information about signals *d*(*t*) and *r*(*t*); 2) Identify the plant model yielding $A^0(z^{-1})$, $B^0(z^{-1})$ and $\alpha(\omega)$; 3) Check whether the control specifications are realistic. If not - redesign the specifications under 1) and/or do a more accurate identification procedure under 2); 4) Solve the system of three Diophantine equations (9), (13), (14) with inequalities (16) and (19) to obtain the controller polynomials *R, S* and *T*.

Challenges and tradeoffs involved in choosing the right set of control specifications are discussed in [1]-[3], and in general represent nontrivial design decisions. A possible choice of characteristic polynomial $K_{pol}(z^{-1})$ [6] is given as

$$K_{pol}(z^{-1}) = \prod_{i=1}^{n}\left(1-b_i z^{-1}\right)^i,\ 0\le b_i\le 0.9\,, \tag{20}$$

which corresponds to a strictly aperiodic closed-loop system step response. Smaller values of n and b_i correspond to higher speed of the system response and lower degree of system robustness. Thus in tuning of n and b_i, it is necessary to start with a certain value of n and smaller values of b_i and then to increase b_i gradually. If for the allowable values of b_i the desired criterions are not satisfied, the value of n should be increased to the next integer and so on.

Another desired criterion is a condition for disturbance suppression (17), which can be reformulated as

$$\left|R(z^{-1})\right|_{z^{-1}=e^{-j\omega T},\,\omega T\in[0,\pi]} \le \left|G_{dy}^{desired}(z^{-1})\right| \left|\frac{K_{pol}(z^{-1})}{A^0(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\,\omega T\in[0,\pi]} . \tag{21}$$

While directly specifying $G_{dy}^{desired}$ is desirable from user perspective, it can be overly constraining in practice due to the inherent tradeoff with the robustness criterion (15), leading often to infeasible optimization problem formulations. Instead, to allow additional degrees of freedom for the optimization problem, while retaining some performance guarantees, we specify the steady-state error behavior utilizing the absorption principle as shown in (13) and (14).

To further illustrate this inherent tradeoff we can write starting from (9), (13), (17),

$$\begin{aligned}\left|K_{pol}\right| &= \left|A^0R+B^0S\right| \le |R|\left|A^0\right| + |S|\left|B^0\right| \\ &\le \left|\Phi_d M_d\right| + \frac{\left|K_{pol}\right|}{\alpha} = \left|G_{dy}\right|\left|K_{pol}\right| + \frac{\left|K_{pol}\right|}{\alpha} \le \left|G_{dy}^{desired}\right|\left|K_{pol}\right| + \frac{\left|K_{pol}\right|}{\alpha}\,,\end{aligned} \tag{22}$$

where the first inequality is the triangle inequality and the second and third follow from imposing constraints in (16) and (21). Simplifying, by diving with non-zero polynomials, we see that (wherever K_{pol} is non-zero, which is almost everywhere as the set of zeros is finite for any polynomial) we must have

$$1 \le \frac{\left|\Phi_d M_d\right|}{\left|K_{pol}\right|} + \frac{1}{\alpha} \le \left|G_{dy}^{desired}\right| + \frac{1}{\alpha} \tag{23}$$

at every discrete frequency. This means that frequencies where our model's uncertainty is high are also critical for disturbance rejection. Also, from (23)

$$\alpha(\omega) \le \frac{1}{1-\left|\dfrac{\Phi_d(z^{-1})M_d(z^{-1})}{K_{pol}(z^{-1})}\right|_{z^{-1}=e^{-j\omega T},\,\omega T\in[0,\pi]}} . \tag{24}$$

Relations (23) and (24) represent a useful check if the given performance and robustness specifications are not simultaneously achievable.

The control design problem given by (9), (13), (14), (16) and (19) can be described as a convex optimization problem. Unlike a pole placement problem [2], this problem formulation enables us to look at a broader range of solutions to Diophantine equations, while still constrained by the robustness stability and noise suppression criterions, leading to potentially better controller performance.

III. Convex Optimization as a Design Tool

In this section we show how the previously described design specifications of the robust RST controller can be transformed into a standard convex program. This step enables us to use readily available optimization software for efficient resolution of design constraints and obtaining the desired RST parameters. In our case the resulting convex programming formulation is a quadratically constrained (QC) feasibility problem.

In order to use the available convex optimization packages, a design problem must be convex and formulated as one of the standard optimization programs [10],[11]. In general, convex optimizations are problems of the following form

$$\begin{aligned}&\min\ f_0(x) \\ &\ s.t.\ f_i(x)\le 0, i=1,\ldots,k\,, \\ &\qquad Ax=b\end{aligned} \tag{25}$$

where the optimization objective (cost), $f_0(x)$, must be a convex function. All the constraints must be convex sets defined either as sub-level sets of convex functions $f_i(x)$, or through linear equality constraints [8].

Our design goal is to determine polynomials *R*, *S* and *T*, given the performance specifications (Φ_r, Φ_d, K_{pol}, $\left|G_{vy}^{desired}\right|$) and the model of the plant ($A^0(z^{-1})$, $B^0(z^{-1})$, $\alpha(\omega)$). The maximum desired order of each polynomial also has to be specified.

For the rest of this section we will represent the RST polynomials with their coefficients, $r=\left(r_1,...,r_{n_R}\right)$, $s=\left(s_1,...,s_{n_S}\right)$ and $t=\left(t_1,...,t_{n_T}\right)$ in the standard representation

$$R(z^{-1})=\sum_{i=0}^{n_R} r_i z^{-i},\quad S(z^{-1})=\sum_{i=0}^{n_S} s_i z^{-i},\quad T(z^{-1})=\sum_{i=0}^{n_T} t_i z^{-i} \qquad (26)$$

where n_R, n_S, n_T are appropriately chosen degrees so that Diophantine equations (9), (13) and (14) can be solved. Note that it is easy to determine the minimum required order of these polynomials by analyzing the orders of predetermined polynomials (Φ_r, Φ_d and K_{pol}) in the Diophantine equations. With this we proceed to analyze the convexity of the design constraints for the robust RST controller. In the most general setting, the inequalities in (16) and (19) do not specify convex sets in polynomial coefficients. To see this we consider these constraints at a fixed digital frequency ω_0. Then all the constraints can be written in the same, abstract, form

$$\left|\sum_{l=0}^{n_P} p_l e^{-jl\omega_0}\right| \le c(\omega_0)\left|\sum_{m=0}^{n_K} k_m e^{-jm\omega_0}\right| \qquad (27)$$

where vector $p\in\{r,s\}$, vector k is the vector of coefficients of $K_{pol}(z^{-1})$ and the positive constant $c(\omega_0)$ depends on the nominal plant model and the uncertainty specification. Obviously, if this inequality is satisfied for some $\tilde{k}$ it is satisfied for $-\tilde{k}$ as well. However, it is not necessarily also satisfied for $\frac{1}{2}\tilde{k}+\frac{1}{2}\left(-\tilde{k}\right)=0$. Thus these constraints are not jointly convex in p and k.

Since we already assumed that the designer has a way of determining a preferable pole placement, if we fix the desired characteristic polynomial $K_{\mathrm{pol}}(z^{-1})$, then the constraints in (16) and (19) become convex. In this case, as the right-hand sides become constants for each discrete frequency, the inequalities are compositions of linear functions of decision variables (the coefficients of the polynomials *R*, *S* and *T*) and the norm inequality defining a sub-level set, which are convex [8].
With this simplifying step, the conditions of robust stability and noise rejection given in relations (16) and (19) become convex, semi-infinite constraints. What this means is that the number of decision (optimization) variables in each constraint is finite (as the *R*, *S* and *T* polynomials are chosen with finite order), but the constraint must be satisfied at infinite number of points (every possible digital frequency). One straightforward technique to deal with this obstacle is to sample each constraint at a certain number of digital frequencies and impose the constraints only in those sampling points [9]. In this way the semi-infinite constraints (16) and (19) become a finite set of simple quadratic constraints on the finite number of decision variables, which can be expressed in the same, abstract, form as

$$\left|\sum_{l=0}^{n_P} p_l e^{-jl\omega_i}\right| \le \tilde{c}(\omega_i), i=1,\ldots,N \qquad (28)$$

for some number of sampling points $\omega_{1,\ldots,N}\in[0,\pi]$. Note that, since $K_{pol}(z^{-1})$ are now fixed polynomials, the right-hand side reduces to a positive constant at each sampling point. The intuition behind this approach is that polynomials are nicely behaved, smooth functions, and we are trying to satisfy this set of semi-infinite constraints on a compact set (set of discrete frequencies). Thus if the sampling of the unity circle is fine enough, satisfying the constraints in a finite number of points will guarantee that they are satisfied on the compact set of interest. Since the design process is not time-critical, we will not pursue a more rigorous qualification of the required sampling grid, as the optimization can always be re-run with a finer grid sampling if we find the conditions of this type to be violated after solving. In this way we obtain one inequality constraint $f_i(x)$, of the formulation (25) in the form of (28). Thus, by requiring that the desired closed loop characteristic polynomial is set by the designer, and by sampling the semi-infinite inequality constraints, we convexify the original model and reduce it to a finite set of convex quadratic inequalities to be handled by the optimization solver.

To ensure the consistency of the model, we must also impose constraints that express the relationship between all the polynomials that factor in (16) and (19), namely the Diophantine relations in (9), (13) and (14). This is a much simpler task. Considering that optimization variables are the coefficients of the polynomials *R*, *S*, *T*, M_r and M_d, and that all other polynomials and their coefficients are fixed, we immediately see that constraints (9), (13) and (14) become a set of linear equations on their coefficients. With polynomial parameterization (26) we can now rewrite (9), (13) and (14) in a more compact form as

$$\begin{pmatrix} T_{a^0} & & & T_{-\varphi_d} & \\ & & T_{b^0} & & T_{\varphi_r} \\ T_{b^0} & T_{a^0} & & & \end{pmatrix}\begin{pmatrix} r \\ s \\ t \\ m_d \\ m_r \end{pmatrix}=\begin{pmatrix} 0 \\ k \\ k \end{pmatrix} \qquad (29)$$

where T_h denotes the Toeplitz matrix for signal *h*. Note that in these equations vectors r, s, t, m_d, m_r contain optimization variables, while vectors a^0, b^0, φ_d, φ_r and k contain only constants. Thus we have defined the matrix $\boldsymbol{A}$ and the vector $\boldsymbol{b}$ in formulation (25).

Finally, we have not included any performance metric in our robust RST model, thus we can trivially assign $f_0(x) = 0$, to complete our optimization program. In general, however, we could define a (convex) function to measure the quality of the controller to be optimized.

From the previous we see that, once the desired performance is specified in a convenient form, synthesis of the RST controller that ensures robust stability to the plant uncertainty, certain level of performance in terms of disturbance and noise suppression and internal consistency of the model can be formulated as a QC optimization problem.

In view of relations (22), (23) and (24) we should note that it might happen that the optimization has no valid solutions for a particular set of design specifications. Unfortunately, due to necessity but not sufficiency of these constraints there is no simple procedure to ensure that the desired controller will exist so that the optimization has a feasible solution. Although a number of obviously infeasible formulations can be filtered-out by using the relations (22)-(24), a significant number of possibly infeasible specifications still exist, leaving this an open research question.

Finally we should note that the described optimization model of the RST controller is by no means a complete one. For example we might want to impose additional constraints to ensure that the solver only allows for stable *R*, *S* and *T* polynomials. This is known to be a non-convex constraint if the sets of all the stable polynomials are considered. However, certain convexifications (that basically reject some solutions in order to construct a convex, inner approximation of the set of stable polynomials) and parameterizations can be used to include such constraints in optimization design flow. Currently the stability of the solution is checked *ex post facto* and improving this aspect of the model is the next logical step in this line of work.

IV. Design Example

The proposed controller synthesis method is shown here for the class of motion control systems with flexible coupling. Our aim is to control the load shaft speed in the presence of torsion vibrations, system parameter variations, disturbance torque, and in the absence of a dedicated load side speed sensor.

Note that many controllers already exist in the field of motion control, but most of them are designed by assuming an ideal, rigid transimission train, adopting $W^0(s) = 1/(J_m + J_l)s$ (J_m, J_l - motor and load inertia) as a simple plant model. As an actual plant model we utilize a model of flexible coupling of motor axle and load [12]. We distinguish the following important data (see Fig. 2) J_m=0.000620 kgm^2, J_l=0.000220 kgm^2, c_s=350 Nm/rad, b_v=0.004 Nms/rad.

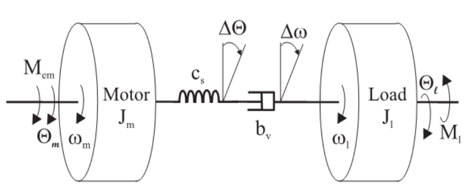

Fig. 2. Flexible coupling of motor axle and load.

The desired close-loop system transfer function is specified by undamped natural frequency ω_n=400 rad/s and relative damping coefficient ζ=0.7. The sample rate is T = 0.5 ms. Adopted absorption polynomials are $\Phi_r = 1 - z^{-1}$, $\Phi_d = (1 - z^{-1})^2$, which fit the step and ramp disturbances.

This formulation of the RST controller design can be solved within minutes, for couple of hundreds sampling points on the unity circle, on standard 2GHz personal computer with 2GB of RAM and running Ubuntu Linux. Our particular implementation of this optimization is done through YALMIP [10] parser and solved with SeDuMi [11] SDP solver.

With the above input specifications the solver's calculation of polynomials *R*, *S* and *T* are: $R(z^{-1})$=0.64-0.77z^{-1}+0.395z^{-2}-0.264z^{-3}, $S(z^{-1})$=0.5-1.1z^{-1}+1.034z^{-2}-0.41z^{-3} and $T(z^{-1})$=0.039z^{-1}. The designed system has met control specifications as illustrated by simulation results in Figs. 3 and 4, for a system reference $\omega_r(t) = 3h(t-0.05)$ rad/s, and a disturbance via load torque $M_l = 1h(t-0.1)$ Nm as in [12]. The disturbance effect d (Fig. 1) for a nominal plant (W^0, $d^0 = W^0 M_l$) is a ramp, while for the real plant in Fig. 2, the torque disturbance manifests itself as the output disturbance $d \equiv W(s)M_l$, which is a ramp with superposed quasi-oscillation at plant resonance frequency (1468 rad/s). The results encompass the time response of the nominal system, and the motor and load speed time responses of realistic plant model structure in Fig. 2. Figure 3 shows that the RST controller has satisfied the robust stability.

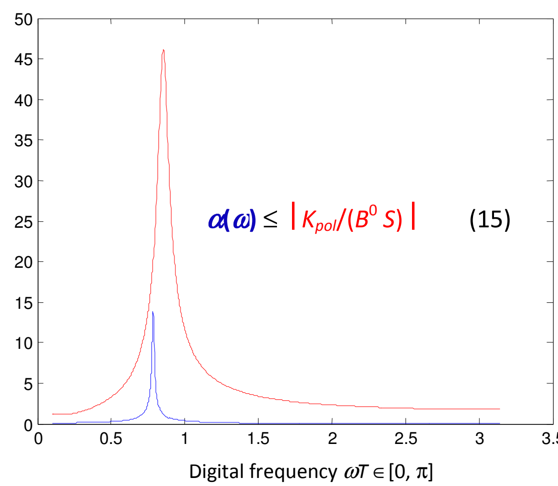

Fig.3. Illustration of the robust stability test.

Multiplicative bound of uncertainties $\alpha(\omega)$ in Fig. 3 is calculated from the realistic and nominal plant models. The successful design of the robust stability and the width of the robustness region are obvious advantages of the proposed design method. The control of the plants with uncertainties at resonant frequencies is a very difficult task in general [6] and control methods for flexible drives mostly do not address these robust stability issues [3], [12].

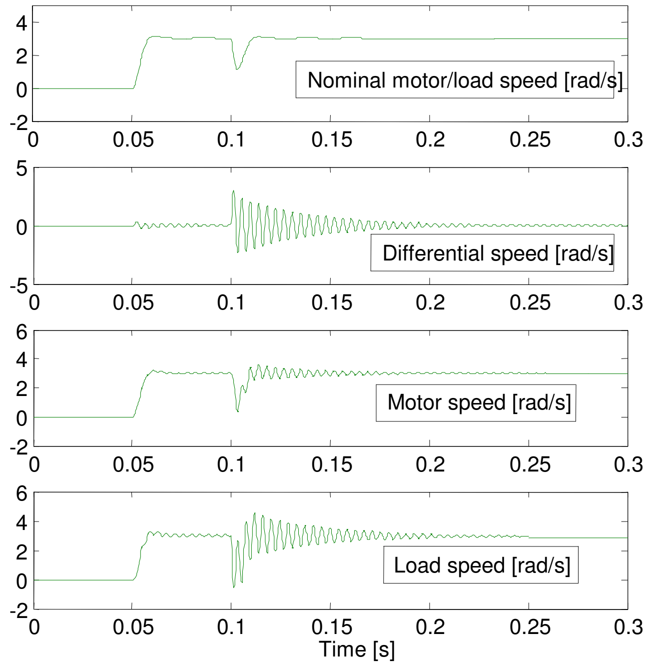

Fig.4. Simulation of the RST controller design efficiency.

V. CONCLUSION

Developing design methodology for an RST controller is a very attractive task since this topology covers the whole space of linear discrete SISO controllers. While some well-known subsets, like PID controller, have been heavily used in industrial applications, other, potentially more-efficient designs have not transitioned yet into industrial settings, mainly due to the lack of a computationally efficient and robust design methodology. The main challenges lie in controller tuning and plant identification procedures and their realization in industrial environment.

In this paper we presented a methodology for design of robust RST controllers, which utilizes the computational efficiency of convex optimization to find the controller designs from a larger set of possible solutions than considered previously in the literature. The convex relaxations allow the search space to be still efficiently constrained by the robust stability and steady-state disturbance and noise rejection specifications, yielding good robustness performance at critical resonant frequencies.

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